



BSP 06

MATHEMATICS

MAXIMUM MARKS: 80

TIME ALLOWED: THREE HOURS

(CANDIDATES ARE ALLOWED ADDITIONAL 15 MINUTES FOR ONLY READING THE PAPER. THEY MUST NOT START WRITING DURING THIS TIME.)

- This Question Paper consists of three sections A, B and C.
- Candidates are required to attempt all questions from Section A and all questions EITHER from Section B OR Section C.
- Section A: Internal choice has been provided in two questions of two marks each, two questions of four marks each and two questions of six marks each.
- Section B: Internal choice has been provided in one question of two marks and one question of four marks.
- Section C: Internal choice has been provided in one question of two marks and one question of four marks.
- All working, including rough work, should be done on the same sheet as, and adjacent to the rest of the answer.
- The intended marks for questions or parts of questions are given in brackets [].
- Mathematical tables and graph papers are provided.

SECTION A - 65 MARKS

Question 1: In subparts (i) to (xi) choose the correct option, and in subparts (xii) to (xv) answer as instructed. [15 × 1 = 15]

(i) If A and B are 3×3 matrices and $AB = I$, then B is:

- (a) unit matrix (b) null matrix (c) singular (d) non-singular

(ii) Let m and n be the order and degree respectively of the differential equation $\left(\frac{d^3y}{dx^3}\right)^2 + \frac{d^2y}{dx^2} = 0$.

The value of $m - n$ is:

- (a) 1 (b) 0 (c) -1 (d) 3

(iii) If $xy = a^2$ (constant a), then $\frac{dy}{dx} =$

- (a) $-x/y$ (b) y/x (c) $-y/x$ (d) y^2

(iv) **Assertion:** For any 3×3 matrix A , $\det(2A) = 2 \det(A)$.

Reason: Multiplying each row of a determinant by 2 multiplies its value by 2^3 .

Which of the following is correct? (a) Both A and R are true, and R is the correct explanation of A.

(b) Both A and R are true, but R is not the correct explanation.

(c) A is true, R is false. (d) A is false, R is true.

(v) Three distinct numbers $x_1 < x_2 < x_3$ are selected at random from $\{1, 2, \dots, 10\}$. What is the probability that $x_2 = 4$ and $x_3 = 7$?

(a) $\frac{1}{28}$ (b) $\frac{1}{56}$ (c) $\frac{1}{40}$ (d) $\frac{3}{28}$

(vi) If $A = \text{diag}(a, b, c)$ is a diagonal matrix, then $A^n =$

(a) $\text{diag}(a^n, b^n, c^n)$ (b) $\text{diag}(a^n, b, c^n)$ (c) $\text{diag}(a, b^n, c)$ (d) $\text{diag}(n a, n b, n c)$

(vii) Statement 1: A function continuous at a point may fail to be differentiable there.

Statement 2: $f(x) = |x|$ is continuous everywhere but not differentiable at $x = 0$.

Which is correct?

(a) S1 true, S2 false (b) S2 true, S1 false (c) Both true (d) Both false

(viii) If $f(x) = kx^2 + 10x + 3$ and $f'(4) = 90$, then $k =$

(a) 5 (b) 7.5 (c) 10 (d) 12

(ix) Statement 1: If a relation R on a set A satisfies $R = R^{-1}$, then R is symmetric.

Statement 2: For R to be symmetric it is necessary that $R = R^{-1}$.

Which is correct?

(a) S1 true, S2 false (b) S2 true, S1 false (c) Both true (d) Both false

(x) **Assertion:** $\cos(\sin^{-1}x) = \sqrt{1-x^2}$.

Reason: $\sin^2\theta + \cos^2\theta = 1$.

Which is correct?

(a) Both A and R true, R explains A (b) Both true, R does not explain A

(c) A true, R false (d) A false, R true

(xi) Given $P(A | B) = 0.4$ and $P(A \cap B) = 0.2$. The probability $P(A' \cap B)$ is:

(a) 0.3 (b) 0.6 (c) 0.8 (d) 0.12

(xii) If A is a 3×3 matrix with $\det A = 4$ and C is the matrix of cofactors of A , find $\det(C^2)$. [1]

(xiii) Let $A = \{a, b, c\}$. If a relation R on A is defined by $R = \{(a, b), (b, a)\}$, classify R (with respect to being reflexive, symmetric, transitive). [1]

(xiv) The function $f: \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = x^2$ is many-to-one. Give a reason. [1]

(xv) In a batch of 3 light bulbs, 2 are defective. The bulbs are tested one by one in random order until both defective ones are found. What is the probability that only two tests are needed? [1]

Question 2 [2]:

(i) If $\log_x y = x^2$, prove that $\frac{dy}{dx} = xy(2\log x + 1)$. (Or)

(ii) Find the value(s) of k for which the function $f(x) = x^3 - 6x^2 + kx$ is increasing for all real x .

Question 3 [2]: Find the point on the curve $y = x^2$ where the tangent is parallel to the chord joining (1,1) and (4,16). [2]

Question 4 [2]: Solve the differential equation $\frac{dy}{dx} = y \tan x$. [2]

Question 5 [2]:

(i) If $\int x^2 \cos(x^3) dx = k \sin(x^3) + C$, find the constant k . [2] (Or)

(ii) Evaluate $\int_0^{\pi/4} \tan x dx$.

Question 6 [2]: Let $f(x) = \tan^{-1}(x/5)$. Compute the value $f(4) + f(6)$. [2]

Question 7 [4]: Solve the equation $\arctan(x) + \arctan(2x) = \frac{\pi}{4}$. [4]

Question 8 [4]: If $y = (A + Bx)e^{-3x}$, prove that $\frac{d^2y}{dx^2} + 6 \frac{dy}{dx} + 9y = 0$. [4]

Question 9 [4]:

(i) Solve $\frac{dy}{dx} = xy$. [4] (Or)

(ii) Solve $(1 + x^2) \frac{dy}{dx} - 2xy = 0$.

Question 10 [4]:

A fair coin is tossed independently by three friends. If one person gets a different result (head or tail) from the other two, that person is declared the loser. If all three results are the same, they toss again.

(a) What is the probability that all three get the same result in one toss round?

(b) What is the probability that they get a different result (one different, two same) in one toss round?

(c) What is the probability that exactly four rounds of tossing are needed to determine a loser?

(Or, answer the parts (a)–(c) for the following situation:)

A student is randomly selected from a group attending three teaching modes: In-person (40% of students), Live online (35%), Recorded lectures (25%). After feedback, 20% of in-person students rated their class "Excellent," 30% of live online did so, and 50% of recorded lecture students did so. A randomly chosen student is found to have rated their class "Excellent."

(a) Define suitable events and express their probabilities.

(b) Using Bayes' theorem, find the probability that this student attended recorded lectures.

(c) Interpret your result: which mode is most likely if a student says "Excellent"? [4]

Question 11 [6]:

Three vendors X, Y, Z sold notebooks, pens, and markers for a school charity. Vendor X sold 10 notebooks, 5 pens, and 15 markers for a total of ₹700. Vendor Y sold 4 notebooks, 10 pens, and 10 markers for ₹480. Vendor Z sold 6 notebooks, 5 pens, and 12 markers for ₹530.

(i) Formulate a system of linear equations for the prices of one notebook, one pen, and one marker.

(ii) Solve this system using a matrix method.

(iii) Find the price of one notebook, one pen, and one marker. [6]

Question 12 [6]: Evaluate the following integrals:

(i) $\int \frac{2x+3}{x^2+4x+3} dx$. [6]



(Or)

(ii) $\int \frac{x}{x^2+2x+2} dx$, [6]

Question 13 [6]:

(i) A closed right circular cylindrical water tank has volume 314π cubic units. If the radius is r and height is h , express h in terms of r . Then express the total surface area S (including top and bottom) in terms of r . Determine the value of r that minimizes S , and find the corresponding height. [6]

(Or)

(ii) A projectile is launched vertically upwards. Its height (in meters) at time t seconds is given by $h(t) = 40t - 4.9t^2$.

- (a) Find the time at which the projectile reaches its maximum height.
- (b) Determine the maximum height attained.
- (c) Find the time when the projectile returns to the ground (height 0). [6]

Question 14 [6]:

A student participates in a quiz competition where she answers three independent questions.

For each question:

- Probability of answering correctly = $\frac{2}{3}$
- Probability of answering incorrectly = $\frac{1}{3}$

She receives:

- ₹100 for each correct answer
- ₹0 for each incorrect answer

Let the random variable X denote the total prize money earned by the student.

(a) Construct the probability distribution table of X .

(b) Find the expected value $E(X)$.

(c) Interpret the expected value in context.

SECTION B – 15 Marks

(Attempt all questions in this section.)

Question 15 [5]: In subparts (i) and (ii) choose the correct option, and in subparts (iii) to (v) answer as instructed. [$5 \times 1 = 5$]

(i) Statement 1: If $\vec{a}$ and $\vec{b}$ are adjacent sides of a parallelogram, then the diagonals are $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$.
Statement 2: If $\vec{a}$ and $\vec{b}$ are diagonals of a parallelogram, then the adjacent sides are $\frac{1}{2}(\vec{a} + \vec{b})$ and $\frac{1}{2}(\vec{a} - \vec{b})$.

Which is correct? (Recall: diagonals formula for parallelogram)

- (a) S1 true, S2 false (b) S2 true, S1 false
(c) Both true (d) Both false

(ii) A plane passes through points $A(1,0,0)$, $B(0,1,0)$, $C(0,0,1)$. The equation of the line through $P(1,2,3)$ normal to that plane is:

- (a) $\vec{r} = (1,2,3) + \lambda(1,1,1)$
(b) $\vec{r} = (1,2,3) + \lambda(1,-1,0)$
(c) $(x-1)/1 = (y-2)/1 = (z-3)/1$
(d) $x=1, y=2, z=3$

(iii) The direction cosines of a line are $(1/a, 1/a, 1/a)$. This implies:

- (a) $a > 0$ (b) $0 < a < 1$ (c) $a = \pm\sqrt{3}$ (d) $a > 2$

(iv) If $\vec{u}$ and $\vec{v}$ are unit vectors with angle θ between them, and $|\vec{u} + \vec{v}| < 1$, then the possible range of θ is:

- (a) $0 < \theta < 60^\circ$ (b) $60^\circ < \theta < 120^\circ$ (c) $120^\circ < \theta < 180^\circ$ (d) $90^\circ < \theta < 180^\circ$

(v) Given $\vec{p} = 3\vec{i} - \vec{j} + 2\vec{k}$ and $\vec{q} = \vec{i} + 4\vec{j} - 2\vec{k}$, find $\vec{p} \cdot \vec{q}$. [1]

Question 16 [2]:

(i) Let $A(0,1,2)$, $B(3,0,1)$, $C(4,3,6)$, and $D(2,3,2)$. Let G be the midpoint of BCD (centroid of triangle BCD). Using this information:

(a) Find the length of $\overline{AG}$.

(b) Find the area of triangle ABC . [2]

(Or)

(ii) Find the values of x for which the angle between the vectors $2x^2\vec{i} + 3x\vec{j} + \vec{k}$ and $\vec{i} - 2\vec{j} + x^2\vec{k}$ is obtuse. [2]

Question 17 [4]:

(i) Given point $A(0,0,0)$. Let points B and C lie on the line $(x-2)/1 = (y-1)/2 = (z+2)/(-1)$. If the length $BC = 4$, find the area of triangle ABC . [4]

(Or)

(ii) Find the equation of the plane that contains the line $(x+1)/3 = (y-2)/(-1) = (z-1)/2$ and the point $P(2,1,0)$. [4]

Question 18 [4]: Find the area bounded by the curve $y = x(4-x)$ and the x -axis, between $x = 0$ and $x = 5$. [4]

SECTION C 15 MARKS

Question -19

In sub-parts(i) and (ii) choose the correct options and in sub-parts (iii) to (v) , answer the questions as instructed.

- (i) The average cost function associated with producing and marketing x units of an item is given by $AC = 3x - 11 + \frac{10}{x}$. Then MC at $x = 2$ is
 (a) 2 (b) 5 (c) 3 (d) 1
- (ii) If $\sigma_x = 5$, $r = -\frac{1}{2}$ and $b_{yx} = -\frac{2}{7}$, then the value of σ_y is:
 (a) $\frac{7}{20}$ (b) $\frac{20}{7}$ (c) $-\frac{20}{7}$ (d) $-\frac{7}{20}$
- (iii) The straight line graph of the linear equation $y = a + bx$, slope will be upward if;
 (a) $a = 0$ (b) $b > 0$ (c) $b < 0$ (d) $b \neq 0$
- (iv) If the marginal revenue function for a commodity is $MR = 9 - 4x^2$, find the demand function.
- (v) Find the marginal cost function (MC), if the cost function is: $C(x) = \frac{x^3}{3} + 5x^2 - 16x + 2$

Question -20

- (i) The average cost function associated with producing and marketing x units of an item is given by, $AC = 2x - 11 + \frac{50}{x}$. Find: (i) the total cost function and marginal cost function (ii) the range of values of the output x , for which AC is increasing.
- (ii) The manufacturing cost of an item is Rs. 3 per item, fixed cost is Rs. 900 and the labour cost is Rs. $\frac{x^2}{100}$ for x items produced. How many items must be produced to have average cost minimum.

Question -21

For observation of pairs (x, y) of the variables X and Y , the following results are obtained.
 $\sum x = 125$, $\sum y = 100$, $\sum x^2 = 1650$, $\sum y^2 = 1500$, $\sum xy = 50$, $n = 25$. find the equation of the line of regression of x on y . Estimate the value of x , if $y = 5$.

Question -22

- (i) Find the line of regression line y on x for the data:

X	1	2	3	4	5
Y	7	6	5	4	3

OR

- (ii) Two tailors P and Q earn Rs. 150 and Rs. 200 per day respectively. P can stitch 6 shirts and 4 trousers a day, while Q can stitch 10 shirts and 4 trousers per day. How many days should each work to produce at least 60 shirts and 32 trousers at minimum labour cost?

